A Simple Functional Form for Proton -$^{7-14}$Be Total Cross Section

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ABSTRACT

Total cross section is calculated for proton scattering on $^{7-14}$Be isotopes using Glauber approach at $30 \leq E_p \leq 2200$ MeV. The calculations show that there is a systematic trend for the obtained cross section results. A simple formula is proposed to give a rapid representation of the total cross sections for $^{7-14}$Be isotopes in the proposed proton energy range. The calculated and predicted cross-section values were compared with the available data in some specific cases.

Keywords: Proton $^{7-14}$Be / Total Cross Section / Glauber approach

INTRODUCTION

The total reaction cross section has been extensively studied theoretically and experimentally (1-10). It is a very important factor for describing the reaction of nucleon-nucleus mode. It can be defined as the summation of all open reaction channels for a projectile with known incident energy that collide a nucleus.

In the framework of Glauber Multiple Scattering Model (GMSM) (11), an extensive global analysis has been used to describe the observed $\sigma_R$ for proton-nucleus collision over a wide range of proton energy (12-17). On the other hand, within the optical-limit approximation (OLA) to the GMSM, the complicated multiple scattering operator is replaced by a simple two-body operator. The basic point of this approximation is to express each partial wave phase shift as an integral along straight line trajectories of quantities involving individual contributions of microscopic collisions weighted by the local matter density. It is found that the predictions of the OLA calculations become less satisfactory at larger scattering angles both as the projectile energy increases and at relatively low energies ($E<100$ MeV) (18-20).

The aim of this work is to calculate the total cross section for $^{7-14}$Be isotopes using Glauber approach, which is introduced for the first time, and to find a systematic mathematical relationship that can connect these results easily to each other. The output results were compared with the available experimental and theoretical data (21).
THEORETICAL MODEL

Within the optical limit approximation of Glauber scattering model, the nuclear phase shift function can be written as (22)

\[ \chi_N^{(1)}(b) = iA \int \rho_1 \bar{r}_1 (I_1(b - \bar{s}_1) \ d\bar{r}_1 \] (1)

The nuclear profile function \( I_1(b - \bar{s}_j) \) for nucleon-nucleon is the two dimensional Fourier transformation of the nucleon-nucleon scattering amplitude,

\[ I_1(b - \bar{s}_j) = \left\{ \frac{1}{2\pi k} \right\} \int d^2 q e^{-i\bar{q} \cdot (b - \bar{s}_j)} f(q) \] (2)

\( f(q) \) is the usual form three-parameters nucleon-nucleon (NN) amplitude which has been applied successfully for many nuclear scattering calculations (2-4). It takes the form,

\[ f(q) = \frac{k \sigma_{NN}(i + \alpha_{NN})}{4\pi} e^{\beta_{NN}q^2} \] (3)

Both \( \sigma_{NN} \), \( \alpha_{NN} \) and \( \beta_{NN} \) are energy dependent total cross section, real-to-imaginary forward \( (q=0) \) scattering amplitude and the slope parameter for NN scattering. \( \rho(r) \) represents the one-body density of the target nucleus. \( A \) is the mass number. The nucleon-nucleus scattering amplitude has the form (22\&25).

\[ F(\bar{q}) = (\frac{ik}{2\pi}) \int d^2 b \left( 1 - e^{i\chi_N(b)} \right) e^{i\bar{q} \cdot b} \] (4)

Where \( b \) is the impact parameter, \( q \) represents the momentum transfer and \( k \) is the nucleon-nucleus center-of-mass momentum.

The total cross-section can be calculated using the optical theorem, where

\[ \sigma_t = \frac{4\pi}{k} Im F(0) \] (5)

Where \( F(0) \) is the nucleon-nucleus scattering amplitude at \( q=0 \).
RESULTS AND DISCUSSION

In this respect the total cross section $p^{7-14}\text{Be}$ in the energy range 30- 2200 MeV were calculated using Eq. (5). We follow our previous work (23) were the densities of the target nuclei are represented by a sum of two Gaussian (24).

$$\rho(r) = \sum_{i=1}^{2} C_i \exp \left[-a_i r^2\right]$$ (6)

The parameters $C_i$ and $a_i$ are taken from (24) within both relativistic mean field (EMF) and motivated effective Lagrangian (E-RMF).

Fig (1) shows a systematic shape of these densities using E-RMF parameters, where these densities gave a plausible agreement in comparison with the experimental data for proton- nucleus reaction cross section (23).

For the Proton- neutron and (pn) and proton- proton (pp) cross sections, we used the following formulae (24-25)

$$\sigma_{pn} = (13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{-4})$$ (7)

$$\sigma_{pp} = (-70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta)$$ (8)

where, $$\beta = \sqrt{1 - \frac{1.0}{\gamma^2}}$$, $$\gamma = \frac{E_{lab}}{931.5} + 1$$

and $$\sigma_{NN} = \frac{Z \sigma_{pp} + N \sigma_{pn}}{A}$$

This formula includes the incident proton energy ($E_{lab}$). It shows that the medium effect is important at intermediate energies and becomes smaller at higher energies but does not vanish (26, 27, 28).

Science, the value of 0.423 fm$^2$ for the slope parameter of NN amplitude gave a reasonable results, in the calculation of the reaction cross section for P- $^3\text{He}$, p-Li, Be and B (23), so in this work we take this value into consideration. Moreover, the values of $\sigma_{NN}$ are taken from (23).

![Fig (1): Density distribution obtained from E_RMF parameters for $^{7-14}\text{Be}$ isotopes (24).](image)
We should mention that; for the output data systematic corrections, we calculated the total reaction cross section $\sigma_R$ for the available parameters of p-$^{27}$Al reaction. A normalization factor $N_o$ was estimated on comparing the experimental and the theoretical values through $30 \leq E_p \geq 2200$MeV energy range. For the total cross section calculation the correction factor $N_o$ was used successfully. Figure (2) shows the calculated reaction cross section in comparison with the available experimental values taken from reference (21).

**Fig (2):** $\sigma_R$ for p-$^{27}$Al reaction as compared with the experimental values of (21).

Fig (3) shows a good agreement for the calculated total nuclear cross section of p-$^9$Be reaction with the experimental results (21). The calculated values were corrected with the $N_o$ factor. It was found that the calculated values give good trend if it is corrected by a factor of $N_o=5\%$. This could be attributed to the poor accuracy determination of the experimental incident proton energy.

**Fig (3):** Calculated $\sigma_t$(mb) for p-$^9$Be reaction, represented by the solid line as compared with the experimental data taken from (21).
Cross Section Systematic Trends for $^{7-14}\text{Be}$ Isotopes

In this section the total cross section for $^{7-14}\text{Be}$ was calculated as well as an expression for this calculation is introduced. Fig (4) represents our calculation and shows the systematic trends of the output results.

![Graph showing systematic trends for $^{7-14}\text{Be}$ isotopes](image)

Fig (4); $\sigma_t(\text{mb})$, Calculated for the first time, for Be isotopes in a wide proton energy range selected from 30-2200 MeV.

In this investigation it was found that, all the cross section values for p-$^{8-14}\text{Be}$ reactions can be predicted from the calculated value of p-$^{7}\text{Be}$, using the simple following formula;

$$\sigma_{tA} = \frac{(A + 1)}{x + 1} \sigma_{t(x)}$$  \hspace{1cm} (9)

Where x in our case can take the values; $7 \leq x \leq (A-1)$

Equation (9) is energy independent and can be used successfully in the energy range $80 \text{ MeV} \leq E_p \leq 2200\text{MeV}$. The calculated $\sigma_t$ for the lower energy part $30 \leq E_p \leq 80\text{MeV}$ was shifted by a factor of 20% from the original values of $^{7}\text{Be}$. Fig (5) and Table (1) show a comparison between the calculated and the fitted values for $^{8&14}\text{Be}$ isotopes.
Fig. (5): Shows the success of using Eq. (9) as a tool for easy production of $\sigma_t$ of $^{8,14}$Be isotopes.

Table (1): Represents the calculated total cross section $\sigma_{tg}$ (mb) using Glauber approach (GA) in Comparison with $\sigma_{te}$ (mb) values predicted from Equation (9).

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Finally it should be concluded that; the introduced formula No. (9) can be used easily as fast prediction method for the calculation of the cross section values of $^{7,14}$Be isotopes if the initial cross section value of $^7$Be is known. Further work has to be done to introduce and test this simple formula for other isotopes group in the periodic table.
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