The Effect of Halo Nuclear Density on the Elastic Scattering of Protons on Halo Nuclei

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ABSTRACT

In the framework of the Glauber optical limit approximation (OLA), the elastic differential cross section \( \frac{d\sigma}{dq^2} \) for protons on halo nuclei has been studied. The results have been displayed at the intermediate energy range of the incident protons. The target nuclei are taken to be one–neutron halo \( (\text{INH}) \) and two–neutron halo \( (\text{IINH}) \). The calculations are carried for Gaussian – Gaussian \( (\rho^{GG}) \), Gaussian – Oscillator \( (\rho^{GO}) \) and Gaussian – 2s \( (\rho^{G2s}) \) densities functions for each considered target. An analytic expression of the phase shift function \( \chi_{\text{OLA}}^{i}; i \equiv GG, GO, G2s \) has been derived. The in – medium and free – medium effects are taken into account and their results are compared for the three considered densities. The obtained results are analyzed and compared with the available experimental data.

Keywords: Proton – Nucleus Collision; Optical Limit Approximation; Halo Nuclei.

INTRODUCTION

Studying the elastic scattering of protons on halo nuclei is of fundamental importance to our understanding of proton induced nuclear reactions. It has opened an exciting channel to look for some crucial issues in context of both nuclear structure and nuclear astrophysics. Also, it has added a new dimension to applications in many fields such as medicine, cosmic ray propagation, detector simulation in particle and high energy physics (¹, ²). Unlike normal (stable) nuclear matter, an interesting feature of some neutron – rich nuclei is a widely extended distribution of loosely bound valence neutrons surrounding a nuclear core, leading to the neutron halo (³–⁵).

Abu – Ibrahim et al have calculated the elastic differential cross section for the scattering of protons on \(^6\text{He}\) at 700 MeV using the complete expansion of the Glauber amplitude (⁶). It has been obtained using a fully microscopic, \( 6 – \) nucleon \( \alpha + n + n – \) cluster wave function. The same model has been applied for the elastic scattering of protons on \(^6\text{He}\) and \(^8\text{He}\) (⁷). But, the harmonic oscillator and LSSM wave functions have been used to describe the density distribution of \(^6\text{He}\) and \(^8\text{He}\), respectively. In addition, the same calculations have been executed but the optical phase shift function is evaluated by Monte – Carlo integration (⁸). This enables to use the most accurate wave functions and calculate the phase shift functions without approximation.

It is found that, the first (leading) term of Glauber Multiple Scattering Model (GMSM) which is known as the optical-limit approximation (OLA), can give fairly good results at relatively lower energies by introducing the coulomb effects (⁹, ¹⁰). This approximation has most commonly been successfully applied for the evaluation of the scattering amplitude and nuclear phase shift function to calculate different nucleon – nucleus cross sections, such as total reaction, total elastic and total cross sections for both stable and exotic nuclei (¹¹–¹⁴).

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The OLA is extensively applied to heavy ion collisions for describing a number of interaction processes over a wide range of energies from near the Coulomb barrier to higher energies \(^{(15)}\). It is a semiclassical model picturing the nuclear collision in the impact parameter representation where the nuclei move along the collision direction in a straight path.

Within OLA, many attempts have been made to describe the nuclear reaction and differential cross sections at high and intermediate energies. M.P. Bush et al have shown that the calculated cross sections for \(^{11}\)Li with different targets at fixed root mean square radii retain a significant sensitivity to higher radial moments of the projectile density \(^{(16)}\). This sensitivity is quite different for light and heavy targets. They have shown that the calculated cross sections depend strongly on the root mean square radius of the density assumed for the projectile nucleus. Shukla et al. have used densities that are obtained from various relativistic mean field formalisms \(^{(17)}\). They have observed that the results found from various densities are similar at smaller scattering angles, whereas a systematic deviation is noticed at large angles. In general, these results agree fairly well with the experimental data. Also, Sharma et al. have compared between relativistic and non – relativistic densities \(^{(18)}\). In general, they have found reasonably good agreement with the experimental data. To get reliable information about nuclear radii and density distributions, Rashdan has attempted to investigate the structure of \(^{16} – ^{20}\)O within the relativistic mean field (RMF) \(^{(19)}\). The reaction cross sections are calculated around 1 GeV using the multiple scattering theory as well as the OLA. The in – medium effect is taken into consideration. The results indicate to a halo structure of \(^{23}\)O.

There is a method to investigate the nuclear matter density distribution of exotic nuclei through the calculations of the proton–nucleus elastic scattering. It was well established before for stable nuclei \(^{(20)}\). This technique is demonstrated to be an effective mean for obtaining accurate and detailed information on the size and the radial shape of halo nuclei. Also, based on Glauber analysis, the p – \(^{6,8}\)He elastic scattering differential cross sections, for small angle, have been performed \(^{(21)}\). Using phenomenological nuclear density distributions with two free parameters, the radii and radial shape of the \(^{6}\)He and \(^{8}\)He nuclei have been determined. In addition, for determining the nuclear matter radii and radial matter distributions, the measured cross sections have been analyzed with the aid of the Glauber multiple-scattering theory \(^{(22)}\). The nuclear matter distribution deduced for \(^{11}\)Li exhibits a very pronounced halo structure. It was found that the matter radius of \(^{11}\)Li being significantly larger than those of the \(^{6,8,^{11}}\)Li isotopes. As an extension, the measured differential cross sections for p – \(^{12,14}\)Be have been analyzed using the Glauber multiple theory \(^{(23)}\).

Several phenomenological nuclear-matter density parameterizations and a sum of Gaussian parameterization have been tested. Alkhazov et al. have discussed the problem of extracting the parameters of nuclear matter and charge distribution in stable and unstable isotopes \(^{(24)}\). They have obtained substantial amount of information on the nuclear radii. Also, some other distribution parameters in light exotic nuclei have been obtained from experiments on intermediate energy nucleus–nucleus collisions.

Different matter density distributions for halo nuclei have been obtained from the measured proton-nucleus elastic scattering cross sections at high incident energies using the Glauber multiple scattering theory \(^{(25} – ^{30})\). It was found that the matter distributions of halo nuclei are significantly more extended than that for their core. For example, in case of \(^{11}\)Li, these studies found that the densities which do not distinguish between neutrons and protons such as one-parameter Gaussian (G) fail to describe the data. Whereas the densities that assume the nucleus to consist of core and halo with different spatial distributions, such as Gaussian-Gaussian (\(\rho^{GG}\)) and Gaussian-oscillator (\(\rho^{GO}\)), fit the data well.

Within the optical limit approximation, the integrated nuclear cross sections for p–\(^{4}\)He are calculated at 721 MeV \(^{(31)}\). Three forms of the nuclear matter density distributions of the target nucleus \(^{4}\)He depending on its cluster structure are considered. Also, using the same approximation, the reaction cross section for halo nucleus – stable nucleus collisions, at intermediate energy, has been studied \(^{(32)}\). The projectile nuclei have been classified into one neutron or two neutron halo nuclei. They can be described by three matter densities with different halo distributions.
In the present work, we will calculate the elastic differential cross sections for the proton scattering with halo targets. The calculations have been performed using OLA model. The range of the studied halo targets is $11 - 29$ of $\text{INH}$ and $\text{IINH}$. Three different density distributions for the halo targets are used. The density functions have the forms $\rho_{GG}$, $\rho_{GO}$ and $\rho_{G\Omega}$. The phase shift functions are analytically obtained for the different studied densities. The in-medium effect is studied. First, we compare obtained results with the available experimental data. Second we intend to study the sensitivity of the elastic differential cross section to the halo part of the target halo nucleus.

### FORMALISM

At high and intermediate energies, the differential cross section $d\sigma/dq^2$ for the elastic scattering of protons from nuclei has been calculated using the optical limit approximation (22) as follows:

$$
\frac{d\sigma}{dq^2} = \frac{\pi}{k^2} |F(q)|^2
$$

where

$$
F(q) = ik \int_0^\infty J_0(qb)\{1 - e^{i[\chi_N(b)+\chi_C(b)]}\}db
$$

is the scattering amplitude and $q^2$ is the four momentum transfer. $\chi_N(b)$ and $\chi_C(b)$ are the nuclear and Coulomb phases, respectively.

When the Coulomb phase is included in the scattering amplitude, divergent terms of the form $e^{2i \ln(kb)}$ generate a problem of integration at large distances. This problem can be sidestepped by adding and subtracting the eikonal phase function for a point like charge (33), then

$$
F(q) = F_{pc}(q) + ik \int_0^\infty J_0(qb)e^{i\chi_{pc}(b)}\{1 - \exp i[\chi_N(b) + \delta\chi_C(b)]\}db
$$

where the scattering amplitude $F_{pc}(q)$ and phase shift function of like point charge $\chi_{pc}\(b\)$ can be represented as,

$$
F_{pc}(q) = -\frac{2k\eta}{q^2} \exp 2i \left[\eta \ln \left(\frac{q}{2k}\right) + arg\Gamma(1 + i\eta)\right],
$$

$$
\chi_{pc}(b) = 2\eta \ln(kb)
$$

where $\eta = \frac{2eZ\hbar}{\hbar v}$ is the Sommerfeld parameter. Also, $\delta\chi_C\(b\)$ has the form,

$$
\delta\chi_C\(b\) = \chi_C\(b\) - \chi_{pc}(b)
$$

The Coulomb phase shift $\chi_C\(b\)$ is taken in an analytical form for the uniformly charged spherical density distribute

$$
\chi_C(b) = \begin{cases} 
2\eta \left[\ln(kR_C) + \ln(1 + \xi) - \frac{1}{3} \xi(3 + \xi^2)\right]; & b \leq R_C \\
\chi_{pc}(b); & b > R_C 
\end{cases}
$$

264
where \( R_C = r_C \left( A_p^{1/3} + A_T^{1/3} \right) \).

In calculations, we take into account the relativistic kinematics correlations by using the respective expressions of speed \( v \) and the c.m. momentum \( k \) \(^{(34)}\)

\[
\begin{align*}
\hbar v &= 197.327 \sqrt{\frac{\delta L_{1/2} (E_L + 2A_p m)}{E_L + A_p m}} \quad (\text{MeV fm}) \\
1 &= \frac{A_T m \sqrt{\delta L_{1/2} (E_L + 2A_p m)}}{(A_p m + A_T m)^{3/2} + 2A_T E_L m} \quad (\text{fm}^{-1})
\end{align*}
\]

where \( E_L \) (in MeV) is the kinetic energy of the incident proton in laboratory system and the unified atomic mass unit \( m = 931.494 \text{ MeV} \).

In the framework of OLA \(^{(35)}\), the nuclear phase shift function \( \chi_N(b) \) for the proton – nucleus scattering can be defined as

\[
\chi_N(b) = i \int \rho_T(r) \Gamma_{NN}(b - s) dr
\]

where \( b \) is the two-dimensional impact vector, \( \Gamma_{NN} \) is the nucleon-nucleon profile function, \( \rho_T(r) \) is the halo target point-density. It is assumed to be normalized to the mass number of the halo nucleus. At the same time, \( s \) is the projection of the position vector \( r \) of the target on the impact plane. The profile function \( \Gamma_{NN} \) in case of the zero range approximation has the form \(^{(25)}\)

\[
\Gamma_{NN}(b_{NN}) = \frac{1}{2} (1 - i \bar{\alpha}_{NN}) \bar{\sigma}_{NN} \delta^{(2)}(b_{NN})
\]

where \( \delta^{(2)}(b_{NN}) \) is the Dirac- \( \delta \) function representing the zero-range approximation and \( \bar{\sigma}_{NN} \) is the free isospin averaged nucleon-nucleon total cross section. It has the form \(^{(36)}\):

\[
\bar{\sigma}_{NN} = \frac{Z_T}{A_T} \sigma_{pp} + \frac{N_T}{A_T} \sigma_{pn}
\]

This parameterization has been taken for energy range \( E_L = 10 - 1000 \text{ MeV} \). Also, \( \bar{\alpha}_{NN} \) is the averaged ratio of real to imaginary parts of the scattering amplitude at \( q = 0 \).

\[
\bar{\alpha}_{NN} = \frac{Z_T \sigma_{pp} \alpha_{pp} + N_T \sigma_{pn} \alpha_{pn}}{Z_T \sigma_{pp} + N_T \sigma_{pn}}
\]

The parameters of \( pp \) (proton – proton) and \( pn \) (proton – neutron) of the profile function \( \Gamma_{NN}(b_{NN}) \) are taken from reference \(^{(37)}\).

The target halo nuclei are assumed to be composed of a core surrounded by a halo of one or more valence neutrons \(^{(38,39)}\). The density of the halo nucleus can be written as \(^{(26)}\):

\[
\rho(r) = N_c \rho_c(r) + N_v \rho_v(r)
\]

where \( \rho_c \) and \( \rho_v \) are the density distributions of the core and valence neutrons, respectively. Also, \( N_c \) and \( N_v \) are the numbers of nucleons in the core and halo parts, respectively.

In the three nuclear matter density distributions of the targets, we have assumed that the core part, in the three densities, is the same and has the Gaussian form

\[
\rho_c(r) = \frac{1}{\pi^{1.5} a^3_c} e^{-r^2/a^2_c}
\]

265
where $a_e = \sqrt{2/3} R_c$ is the diffuseness parameter of the core with $R_c$ is the RMS radius of the core part. On the other side, the matter density of the halo part in the target nuclei, $\rho_e(r)$, has three different forms. The first form $\rho_e^G(r)$ describes the halo part as Gaussian distribution \(^{(23,24)}\):

$$\rho_e^G(r) = \frac{1}{\pi^{1.5}a_G^3}e^{-r^2/a_G^2}$$  \hspace{1cm} (16)

where $a_G = \sqrt{2/3} R_v$ is the diffuseness parameter of the halo part with $R_v$ is the RMS radius of the halo part. The radii $R_c$ and $R_v$ are related to the matter radius of the halo nucleus $R_p$ by the following equation

$$R_T = \sqrt{\frac{N_c R_c^2 + N_v R_v^2}{A_p}}$$  \hspace{1cm} (17)

In the second one, a $lp$-shell harmonic oscillator concept is used for the density of the valence neutron(s). This second form is described as \(^{(23,24)}\):

$$\rho_e^O(r) = \frac{1}{1.5 \pi^{1.5}a_O^3}r^2e^{-r^2/a_O^2}$$  \hspace{1cm} (18)

where $a_O = \sqrt{2/5} R_v$. Finally, in case of the G2s density form, it is assumed that the valence neutrons are in 2S state \(^{(40,41)}\) and the halo distribution takes the form:

$$\rho_e^{2S}(r) = \frac{1}{1.5 \pi^{1.5}a_{2S}^3} \left( \frac{r^2}{a_{2S}^2} - \frac{3}{2} \right)^2 e^{-r^2/a_{2S}^2}$$  \hspace{1cm} (19)

where $a_{2S} = \sqrt{2/7} R_v$.

These different projectile densities were used before to study the sensitivity of the reaction cross section of exotic halo nucleus – nucleus scattering \(^{(36)}\). It has been concluded that the ratio $R_v/R_c$ plays an essential role in the behaviour of the densities and, consequently, in the calculations of the reaction cross-section. Using equations (11 – 19), one can get according to equation (10), the explicit formula of the phase shift functions $\chi_G^{G_O}(b), \chi^{G_O}(b)$ and $\chi^{G_{2S}}(b)$ in the case of GG, GO and G2s, respectively:

$$\chi_G^{G_O}(b) = \bar{\sigma}_{NN}(i + \bar{a}_{NN}) \left\{ \frac{N_c}{\pi a_c^2} e^{-b^2/a_c^2} + \frac{N_v}{3\pi a_O^2} (a_O^2 + 2b^2) e^{-b^2/a_O^2} \right\}$$  \hspace{1cm} (20)

$$\chi^{G_O}(b) = \bar{\sigma}_{NN}(i + \bar{a}_{NN}) \left\{ \frac{N_c}{\pi a_c^2} e^{-b^2/a_c^2} + \frac{N_v}{3\pi a_O^2} (a_O^2 + 2b^2) e^{-b^2/a_O^2} \right\}$$  \hspace{1cm} (21)

$$\chi^{G_{2S}}(b) = \bar{\sigma}_{NN}(i + \bar{a}_{NN}) \left\{ \frac{N_c}{\pi a_c^2} e^{-b^2/a_c^2} + \frac{N_v}{3\pi a_{2S}^2} (a_{2S}^2 + 4a_{2S}^2 b^2 + b^4) e^{-b^2/a_{2S}^2} \right\}$$  \hspace{1cm} (22)

To take into consideration the in – medium effect, the nucleon-nucleon cross sections: $\sigma_{pp}$ and $\sigma_{pn}$ will be replaced by the formula:

$$\sigma_{pp}^m = \sigma_{pp} f_m(pp) \hspace{1cm} and \hspace{1cm} \sigma_{pn}^m = \sigma_{pn} f_m(pn)$$  \hspace{1cm} (22)

respectively, where $f_m(nn)$ and $f_m(pn)$ are the in – medium factors \(^{(42)}\).
Consequently, $\bar{\sigma}_{NN}$ will be replaced by $\bar{\sigma}_{N\bar{N}}$ when the in-medium effect is taken into account. As shown in equation (23), if $\bar{\rho} = 0$, one gets $f_m(pn) = f_m(pp) = 1$. This situation can be referred to as the free nucleon-nucleon collision. We have considered $\bar{\rho}$ to be written as:

$$\bar{\rho} = \frac{1}{A_T} [N_c\rho_c(0) + N_o\rho_v(0)].$$

In this equation, $\rho_c(0)$ is the saturated matter density of the target and the core part of the target according to equations (15). Moreover, it is important to mention that $\rho_v(0)$ takes the following different forms: $\rho_v^G(0)$, $\rho_v^0(0)$ and $\rho_v^{2S}(0)$ according to equations (16), (18) and (19), respectively.

## RESULTS AND DISCUSSION

The elastic scattering of protons off neutron halo nuclei has been studied at intermediate energies. The results are displayed in figures (1) – (5). These figures show the theoretical differential cross section ($d\sigma/dq^2$) displayed as a function of the four momentum transfer $q^2$(GeV/c)$^2$. Calculations have been done in the framework of OLA. The mass number of the halo targets ranges from $11 \rightarrow 29$. The halo nuclei have been divided into INH and IINH. The density distributions of these halo nuclei are Gaussian – Gaussian ($\rho^G$), Gaussian – Oscillator ($\rho^0$) and Gaussian – 2s ($\rho^{2S}$) (eq.’s 14 – 19). The root mean square radii (RMS) are taken from ref. [3, 37]. In table (1), we have referred to the structure of the target halo nuclei. The RMS of the target ($R_m$), the core part ($R_c$) and the halo part ($R_v$) referred to in the 3rd, 4th, and 5th columns, respectively. In the last column, we have stated the ratio between the RMS radii of the halo to the core parts ($R_v/R_c$).

### Table (1): The core-halo structure and RMS radii of target nuclei (3, 37)

<table>
<thead>
<tr>
<th>Types of Targets</th>
<th>Core-halo structure</th>
<th>$R_p$ (fm)</th>
<th>$R_c$ (fm)</th>
<th>$R_v$ (fm)</th>
<th>$R_v/R_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INH</strong></td>
<td>$^{11}$Be $\rightarrow$ $^{10}$Be + n</td>
<td>2.73</td>
<td>2.30</td>
<td>5.39</td>
<td>2.3434</td>
</tr>
<tr>
<td></td>
<td>$^{15}$C $\rightarrow$ $^{14}$C + n</td>
<td>2.40</td>
<td>2.30</td>
<td>3.10</td>
<td>1.3478</td>
</tr>
<tr>
<td></td>
<td>$^{17}$C $\rightarrow$ $^{16}$C + n</td>
<td>2.72</td>
<td>2.70</td>
<td>3.02</td>
<td>1.1185</td>
</tr>
<tr>
<td></td>
<td>$^{19}$C $\rightarrow$ $^{18}$C + n</td>
<td>3.13</td>
<td>2.82</td>
<td>6.56</td>
<td>2.3262</td>
</tr>
<tr>
<td></td>
<td>$^{22}$N $\rightarrow$ $^{21}$N + n</td>
<td>3.07</td>
<td>2.75</td>
<td>6.97</td>
<td>2.5345</td>
</tr>
<tr>
<td></td>
<td>$^{23}$O $\rightarrow$ $^{22}$O + n</td>
<td>3.20</td>
<td>2.88</td>
<td>7.28</td>
<td>2.5278</td>
</tr>
<tr>
<td></td>
<td>$^{24}$F $\rightarrow$ $^{23}$F + n</td>
<td>3.03</td>
<td>2.79</td>
<td>6.43</td>
<td>2.3047</td>
</tr>
<tr>
<td></td>
<td>$^{26}$F $\rightarrow$ $^{25}$F + n</td>
<td>3.23</td>
<td>3.12</td>
<td>5.28</td>
<td>1.6923</td>
</tr>
<tr>
<td></td>
<td>$^{29}$Ne $\rightarrow$ $^{28}$Ne + n</td>
<td>2.99</td>
<td>2.92</td>
<td>4.50</td>
<td>1.5411</td>
</tr>
<tr>
<td><strong>IINH</strong></td>
<td>$^{11}$Li $\rightarrow$ $^9$Li + 2n</td>
<td>3.12</td>
<td>2.50</td>
<td>5.04</td>
<td>2.0160</td>
</tr>
<tr>
<td></td>
<td>$^{14}$Be $\rightarrow$ $^{12}$Be + 2n</td>
<td>3.16</td>
<td>2.59</td>
<td>5.45</td>
<td>2.1042</td>
</tr>
<tr>
<td></td>
<td>$^{17}$B $\rightarrow$ $^{15}$B + 2n</td>
<td>3.00</td>
<td>2.47</td>
<td>5.55</td>
<td>2.2470</td>
</tr>
<tr>
<td></td>
<td>$^{19}$B $\rightarrow$ $^{17}$B + 2n</td>
<td>3.11</td>
<td>3.00</td>
<td>3.90</td>
<td>1.3000</td>
</tr>
<tr>
<td></td>
<td>$^{22}$C $\rightarrow$ $^{20}$C + 2n</td>
<td>3.58 $^{37}$</td>
<td>2.99</td>
<td>7.18</td>
<td>2.40</td>
</tr>
</tbody>
</table>
The explicit form of OLA phase shift has been calculated analytically. The zero range approximation is taken into consideration. In addition, the in-medium effect has been inserted in the proton–proton and proton–neutron cross sections \( \sigma_{pp} \) and \( \sigma_{pn} \), respectively. The Coulomb effect is taken into account by using the analytical form of the uniformly charged spherical density distribution.

First, in figure (1), we have compared between the theoretical results of \( \frac{d\sigma}{dq^2} \) for \( ^6\text{He} \), \( ^{11}\text{Li} \) and \( ^{14}\text{Be} \) at 721, 697 and 703 MeV, respectively. The in-medium and free-medium effects have been considered. Due to the lack of the available experimental data, the calculations have been done for \( q^2: 0 \rightarrow 0.05\text{(GeV/c)}^2 \). This is to test the applicability of the current model to describe the elastic scattering of protons off the studied halo nuclei. The experimental data are taken from ref. (21–23).

For an objective discussion for the calculated results, the \( \chi^2 \) parameter is calculated using the following expression:

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{(d\sigma_i/dq^2)_{\text{theo}} - (d\sigma_i/dq^2)_{\text{exp}}}{(d\sigma_i/dq^2)_{\text{exp}}} \right|^2
\]

where \( (d\sigma_i/dq^2)_{\text{theo}} \) are the calculated differential cross section, \( (d\sigma_i/dq^2)_{\text{exp}} \) are the measured ones and \( N \) refers to the number of points. The \( \chi^2 \) parameter is calculated for each interaction and the results are tabulated in table (2).
Table (2): The $\chi^2$-fitting parameter for the elastic proton – halo nucleus scattering.

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2_{GG}(f)$</th>
<th>$\chi^2_{GG}(m)$</th>
<th>$\chi^2_{GO}(f)$</th>
<th>$\chi^2_{GO}(M)$</th>
<th>$\chi^2_{G2S}(f)$</th>
<th>$\chi^2_{G2S}(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6$He</td>
<td>8.3955</td>
<td>0.6223</td>
<td>12.1437</td>
<td>0.5037</td>
<td>9.07871</td>
<td>0.5199</td>
</tr>
<tr>
<td>$^{11}$Li</td>
<td>148.022</td>
<td>56.4655</td>
<td>150.856</td>
<td>52.5085</td>
<td>115.5668</td>
<td>60.4206</td>
</tr>
<tr>
<td>$^{14}$Be</td>
<td>325.046</td>
<td>78.0905</td>
<td>332.885</td>
<td>73.0226</td>
<td>277.338</td>
<td>82.3717</td>
</tr>
</tbody>
</table>

The minimum value of the $\chi^2$ parameter shows the best theoretical results of $(d\sigma/dq^2)$ of the different cases. It enables us to have an overall view on the behavior of each graph. The 1st column $\chi^2_{GG}(f)$ is the $\chi^2$ parameter for the results obtained by $\rho^{GG}$ and in case of free – medium consideration. The 2nd column ($\chi^2_{GG}(m)$) is the same as the 1st column except that we have inserted the in – medium effect. The 3rd and 5th columns are the same as the 1st column but for $\rho^{GO}$ and $\rho^{G2s}$, respectively. Also, the 4th and 6th columns are the same as the 2nd column but for $\rho^{GO}$ and $\rho^{G2s}$, respectively.

Fig. (2): The elastic differential cross sections for the scattering of protons with INH targets ($^{11}$Be,$^{15,17,19}$C,$^{22}$N,$^{23}$O and $^{24,26}$F and $^{29}$Ne) at 800 MeV. The black, red and blue curves stand for the target matter densities GG, GO and G2s.
In general, in case of the three considered interactions, (figure (1) and table (2)), it is clear that the results in case of the in – medium effect gives better fit with the experimental data than the free – medium consideration. Moreover, it is observable that \( \chi^2_{GO(m)} \) is the minimum in case of the three interactions. This refers to that \( (d\sigma/dq^2)_{GO} \) in case of in – medium effect gives the best description for the proton scattering with the studied halo nuclei.

In addition, we intend to study the sensitivity of the elastic scattering to the halo part of the density functions. The theoretical results of \( d\sigma/dq^2 \) have been displayed as a function of \( q^2 \) at 800 MeV. The results have been presented in case of INH and IINH in figures (2) and (3), respectively.

The calculations have been executed taking into account the in – medium effect. We have compared between the differential cross sections \( d\sigma/dq^2 \) in case of \( \rho^{GG}, \rho^{GO} \) and \( \rho^{G2S} \) (black, red and blue curves, respectively).

**Fig. (3):** The same as figure (2) but for IINH targets \((^{11}\text{Li}, ^{14}\text{Be}, ^{17&19}\text{B and } ^{22}\text{C})\)

In case of INH, (figure (2)), the shape of the results in case of \( R_v/R_c > 2 \) differs from the shape of the results in case \( R_v/R_c < 2 \). From figures (1 – b), (1 – c), (1 – h) and (1 – i), there are certain properties for the results in case of \( R_v/R_c < 2 \). There is no oscillation all over the range of the results except that there is one noticeable minimum. The results in case of \( \rho^{GG} \) and \( \rho^{GO} \) are nearly coincident but the results in case of \( \rho^{G2S} \) are different.

On the other side, figures (1 – a), (1 – d), (1 – e), (1 – f) and (1 – g) displayed the results of \( ^{11}\text{Be}, ^{19}\text{C}, ^{22}\text{N}, ^{23}\text{O} \) and \( ^{24}\text{F} \), where \( R_v/R_c > 2 \). In these figures the oscillations are clear at \( q^2 > 0.2(GeV/c)^2 \). As an exception, in case of \( ^{11}\text{Be} \), the oscillations start to appear at \( q^2 > 0.5(GeV/c)^2 \). Also, there is no clear differences between the results at small \( q^2 \) but the differences start to appear at large \( q^2 \).
The same features can be observed in case of INH (figure (3)). By comparing the interactions p+\textsuperscript{11}Li, \textsuperscript{14}Be, \textsuperscript{17}B and \textsuperscript{23}C (figures (3 - a), (3 - b), (3 - c) and (3 - d), respectively) with p+\textsuperscript{19}B (figures (3 - d)), we can note a difference in the shape of $d\sigma/dq^2$. The difference in the shape of $d\sigma/dq^2$ is related to the value of $R_q/R_c$. In case of p+\textsuperscript{19}B, where $R_q/R_c < 2$, there is no oscillations except that there is one pronounced minimum at small $q^2$. In addition, $d\sigma/dq^2$ in case of $\rho^GG$ and $\rho^GO$ are coincident while $d\sigma/dq^2$ in case of $\rho^G2S$ does not agree with them. For the other interactions, where $R_q/R_c > 2$, the oscillations are clear with several pronounced minima. The differences between the results are clearer than the case of INH.

**Fig. (4):** The ratio between the elastic differential cross sections for different densities in case of INH. The black, red and blue curves stand for $\lambda^G_\text{GG}/\text{GO}$, $\lambda^G_\text{GG/G2S}$ and $\lambda^G_\text{GO/G2S}$, respectively.

To show up the differences between the results for the studied densities, the ratios $\lambda^G_\text{GG/GO} \equiv (d\sigma/dq^2)_\text{GG}/(d\sigma/dq^2)_\text{GO}$, $\lambda^G_\text{GG/G2S} \equiv (d\sigma/dq^2)_\text{GG}/(d\sigma/dq^2)_2\text{S}$ and $\lambda^G_\text{GO/G2S} \equiv (d\sigma/dq^2)_\text{GO}/(d\sigma/dq^2)_2\text{S}$ are presented in figures (4) and (5) for INH and INH, respectively. The deflection of the ratios around the unity refers to the differences between the results for the indicated densities.

From figure (4), the shape of the ratios in case of $R_q/R_c > 2$ differs from the shape of the ratios in case of $R_q/R_c < 2$. In case of $R_q/R_c > 2$, the ratios between the results for the studied densities are nearly the same for $q^2 < 0.6(\text{GeV}/c)^2$ except in light fluctuations. This appears in case of p+\textsuperscript{11}Be, \textsuperscript{15}C, \textsuperscript{22}N, \textsuperscript{25}O and \textsuperscript{26}F (figures 1 - a), (1 - d), (1 - e), (1 - f) and (1 - g), respectively. But for $q^2 > 0.6(\text{GeV}/c)^2$, there are irregular hesitation because of the differences appear in the
oscillations at large $q^2$. But in case of $R_v/R_c < 2$, there are different notes. The black line ($\lambda_{GG/GO}$) which refers to the change between the results of GG and GO results oscillates smoothly around the unity. This means that $\rho_{GG}^{GG}$ and $\rho_{GO}^{GO}$ nearly give the same results. Whereas, the red and blue lines have similar changes either more or less than the unity.

In figure (5), the notes in case of IINH are the same as INH. From table (1), the nuclei $^{11}$Li, $^{14}$Be, $^{17}$B and $^{22}$C have the property $R_v/R_c > 2$. In case of the proton interaction with these nuclei, the ratio $\lambda_{GG/GO}$ is larger than the other ratios. Also, the ratios $\lambda_{GG/G2S}$ and $\lambda_{GO/G2S}$ are nearly equal at small $q^2$. For larger $q^2$, the ratios oscillate irregularly around the unity. In case of $p^{+19}$B (figure (5-d)), the target halo nucleus has the property $R_v/R_c > 2$. It is observed that $\lambda_{GG/GO}$ nearly equal the unity all over the range of $q^2$. The other two ratios $\lambda_{GG/G2S}$ and $\lambda_{GO/G2S}$ does not equal to $\lambda_{GG/GO}$. This reflects that $(d\sigma/dq^2)_{GG}$ and $(d\sigma/dq^2)_{GO}$ are the same but $(d\sigma/dq^2)_{G2S}$ does not equal them.

CONCLUSION

First, we can conclude that the in – medium effect is important in enhancing the results to fit with the experimental data. Second, the sensitivity of the elastic differential cross section for proton scattering with halo nuclei is strongly connected to the ratio of the halo radius to the core radius $(R_v/R_c)$. The critical value of this ratio equals 2. If $R_v/R_c > 2$, $d\sigma/dq^2$ has nearly the same shape in case of the three studied densities. But if $R_v/R_c < 2$, $(d\sigma/dq^2)_{GG}$ and $(d\sigma/dq^2)_{GO}$ have the same shape while $(d\sigma/dq^2)_{G2S}$ has another shape. This means that $\rho^{G2S}$ becomes different from the other two densities if $R_v/R_c < 2$. In addition, the oscillations that appear in $d\sigma/dq^2$ in case of $R_v/R_c > 2$ may be due to that the valence neutrons are far from the core part.
REFERENCES