Yrast Bands in Even-Mass Pt and Hg Isotopes

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ABSTRACT

A simple semi-empirical model based on the vibrational model is used to investigate the yrast bands of 178-180Pt and 182-198Hg isotopes. Additionally, the model overcomes the effect of the shape coexistence and shape transitions. This model gives a good fit to the observed small spacing between the 10⁺ and 12⁺ levels of 190Pt and the closely spaced triplet (8⁺, 10⁺ and 12⁺) structure in the energy spectra of 188-198Hg isotopes as compared with the other main models.

Keywords: Even Pt and Hg/ Yrast bands/Vibrational Model

INTRODUCTION

The appearance of shape coexistence in nuclei has attracted a lot of attention in recent decades[1-3] and compelling evidence has been obtained, in particular, at and very near to proton or neutron closed shells. The Pb region takes a prominent position because in addition to specific shell-model excitations established close to the N = 126 neutron shell closure, collective excitations have been observed for the neutron-deficient nuclei[4]. The review paper of Julin et al.[5] gives an extensive overview of the rich variety in nuclear excitation modes for both the Pb, Hg, Pt and, Po, Rn nuclei. Early calculations using a deformed Woods–Saxon potential in order to explore the nuclear energy surfaces as a function of the quadrupole deformation variables showed a consistent picture pointing out the presence of oblate and prolate energy minima. More recently, mean-field calculations are going beyond the static part, including dynamical effects using the Generator Coordinate Method (GCM)[8].

Custen and Cizewski,[9] have been described the Pt nuclei by the (IBM–1) model as O(6)→SU(3) transition. However, the energy levels could not be fitted well. García-Ramos and Heyde,[10] have been studied the Pt nuclei, by the IBM calculations considering the 4 proton holes and the number of valence neutrons with the boson model approximation.

In the heavier 190Pt, it is observed that the energy spacing between the 10⁺ and 12⁺ levels is rather small. A possible explanation of this anomaly is due to the rotational-aligned bands of (νi13/2) or (πh11/2) character which were suggested to intersect the ground-state band and the first crossing is responsible for the observed small spacing[12,13].

The very neutron-deficient Hg isotopes have been the focus of attention ever since the discovery by Bonn et al.[14]. The experimental situation showed that throughout nearly the whole chain there exists (at least) one band which is dominated by near-spherical vibrational structures. This should have been expected since the charge number (Z=80) is near a closed shell (Z=82). These near spherical states are remarkably constant in energy for A=188-196.

The lower mass isotopes (A=182-188), show a surprising well-established coexistence of a near spherical ground state oblate band (β=0.12) and a well-deformed prolate band (β=0.25) the latter is rising in energy with increasing N[15,16]. For the higher isotopes (A=190-198), there is no low-lying prolate band, but a closely spaced triplet 8⁺, 10⁺ and 12⁺ intervenes between the ground oblate and the ν13/22 aligned band.

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Barfield et al., (1983)\(^{(17)}\) has been treated the prolate and oblate shape coexistence in the frame work of the interacting boson model IBM-2. A unified description of Hg isotopes where up to two and three quasiparticle states were involved for even and odd mass isotopes of Hg respectively was previously presented by Kuyucak et al.,(1984)\(^{(18)}\). The interacting-boson-model approach including four quasiparticles have been applied to \(^{194}\)Hg by Iachello and Vretenar (1991)\(^{(19)}\) and a suitable results have been obtained.

Additionally, the shape coexistence and shape transitions in Hg isotopes have been discussed by Troltenier et al. (1991)\(^{(20)}\) by using the General Collective Model. The spectral anomalies of even-even Hg isotopes have been studied in the interacting-boson-approximation-plus-fermion-pair model by Hsieh et al.(1992)\(^{(21)}\). In the same work, both the shape evolution and the observed \((8^+, 10^+, 12^+)\) triplet states can be reproduced with moderate and/or notable deviations as indicated in Figure (5) in their work. Hsieh et al.\(^{(21)}\) interpreted this triplet structure by the \((i_{13/2})^2\) band crossing and the saturation of \(T=I\) fermion-pair interaction at the high-j end. Iachello and Vretenar\(^{(20)}\) have shown that the wave function of \(^{194}\)Hg can be interpreted as two quasiparticle states weakly coupled with the boson core. So, the dominant configurations for these triplet states should be \(I = 8,10,12 \ (i_{13/2})^2\) fermion-pair states coupled with \(L_b=0\) boson states. Since the two-body matrix elements for the \((i_{13/2})^2\) fermion pair with \(I = 8,10\) and 12 are very close, this produces the triplet structure of the \(I = 8,10\) and 12 states. Also, all the previous theoretical investigations of the yrast band in even-even Hg isotopes lead to the use of multiparameter complicated models. So, it seems necessary to reinvestigate the energy levels in even-even Hg isotopes using a rather simple model. Therefore, in the present work the yrast bands of \(^{178-190}\)Pt and \(^{188-198}\)Hg isotopes have been interpreted on the basis of a semi-empirical model based on the vibrational model. The model not only reasonably describes the level energies but its validity extends to reproduce nicely the closely spaced triplet \((8^+, 10^+, 12^+)\) structure in the energy spectra of \(^{188-198}\)Hg.

**MODEL DESCRIPTION**

To describe the main properties of the low-energy spectra for nuclei, collective models are quite convenient and effective. One group of models is based on a classical picture where the nucleus is described by a liquid drop or a similar object which has a definite surface.

The collective states are then given by vibrations and rotations of this drop\(^{(22)}\). Accordingly the nuclei can be classified as closed-shell, vibrational and rotational nuclei. In each region certain suitable nuclear models have been proposed which have been explained the systematic trends of the nuclear properties of the excited states. In the view point of collective vibrational excitations, the nucleus is considered to be a dynamic system which can perform small oscillations about the equilibrium shape. The oscillation can be analyzed in terms of normal modes.

According to the liquid-drop model, the nucleus is considered as an incompressible liquid drop with a sharp surface, and the surface oscillations come about as a result of the variation of the deformation parameter \(a_\mu\) which determine the nuclear shape. Where \(\mu\) takes the values \(-\lambda\) to \(+\lambda\) .

In the harmonic approximation, the excitation energy is given by\(^{(24)}\)

\[
E = E(n_\lambda) - E(n_\lambda = 0) = n_\lambda \omega_\lambda 
\]

Where \(n_\lambda\) is the number of quanta or phonons

\[
n_\lambda = \sum_\mu n_{\lambda\mu} \quad (2)
\]

And \(\lambda\) is the angular momentum of a phonon in the state \(\lambda\), its \(z\)-component is \(\mu\). For \(n_\lambda = 2\), the normalised states have

\[
I = 0, 2, 4, \ldots, 2\lambda \quad (3)
\]

and \(\omega_\lambda\) is the classical frequency of oscillation, it is given by

\[
\omega_\lambda = \sqrt{\frac{\hbar}{\mu B_\lambda}} \quad (4)
\]
Where $C_\lambda$ is the deformability coefficient and has been evaluated by Ring and Schuck$^{(25)}$ as follows:

$$C_\lambda = (\lambda - 1)(\lambda + 2)R_0^2a_s - \frac{3(\lambda-1)2^2e^2}{2\pi(2\lambda+1)R_0}$$

(5)

where $a_s$ denotes the surface energy constant with a value of 18.56MeV, and $B_\lambda$ is the moment of inertia of the nucleus with respect to changes in deformation and is given by

$$B_\lambda = \frac{3AmR_0^2}{4\pi\lambda}$$

(6)

Where $m$ is the nucleon mass and $R_0=1.25A^{1/3}$

In general, equation (1) can be expressed as

$$E=A\omega l$$

(7)

Where, $A$ can be called as a first order phonon parameter.

It is well known that eq. (7) can't express the yrast bands by any way. If we take into account that there are several interactions such as phonon-phonon interaction, phonon-particle interaction etc., Boher and Mottelson,$^{(24)}$ showed that at higher excitation energies, a number of other vibrational modes have been identified.

This work, shows that the yrast bands for even-even Hg and Pt isotopes with mass numbers $182 \leq A \leq 190$ and $Z$ lies near the magic number 82 can be expressed by a simplified expression based on the shell model with minimum fitting parameter to describe the spectra variation from light isotopes to heavy ones, including shape coexistence. It is found that, the yrast bands for Hg and Pt isotopes can be described in a good manner if we take $I=2\lambda$ values.

From the previous discussion the energy of states $E(I)$ can be expressed as an expansion in a powers of $\omega_l$ as follows,

$$E = A\omega_1 + B\omega_2^2 + C\omega_3^3 + D\omega_4^4 + \cdots$$

(8)

Where, $B$, $C$ and $D$ are higher order parameters.

The even power terms in the previous expression are comparable to the so-called Harris expansion$^{(26)}$ for rotational spectra. The difference is that $\omega$ in Harris expansion is the rotational frequency rather than the frequency of oscillation $\omega_l$. The odd power terms in Eq.(8) could describe the residual interaction coming from band mixing, shape coexistence and shape transitions or so on. Furthermore, Eq.(8) is equivalent to the extended variable moment of inertia model to high spins by Anagnostatos$^{(27)}$ based on the article given by Das and Banerjee.$^{(28)}$ In that work, the energy of the state of an even-even nucleus is in the form:

$$E = C_2(\varphi - \varphi_0)^2 + C_3(\varphi - \varphi_0)^3 + C_4(\varphi - \varphi_0)^4 + \frac{l(l+1)}{2\varphi^2}$$

(9)

Where $C_2$, $C_3$, $C_4$ and $\varphi_0$ are the four parameters of the model.

RESULTS AND DISCUSSION

In the present work, the yrast bands in $^{182-198}$Hg and $^{178-188}$Pt isotopes are well reproduced by using the modified model with small number of parameters (Eq.8). Table(1), includes the four parameters $A$, $B$, $C$ and $D$ which are calculated by using the least squares fit to the experimental data$^{(26-32)}$.

Fig.(1) represents the relation between the angular momentum $I$ and $\Delta E(I)=E(I)-E(I-2)$ for the yrast states of $^{178-190}$Pt isotopes. From the figure it is obviously clear that there is a good agreement between the calculated and the experimental data. Additionally, the small energy spacing between the $10^+$ and $12^+$ levels in the $^{190}$Pt isotope is very well predicted.
Table (1): The fitting parameter in MeV of the model of Eq. (8).

<table>
<thead>
<tr>
<th>Element</th>
<th>A×10^2</th>
<th>B×10^3</th>
<th>C×10^4</th>
<th>D×10^5</th>
</tr>
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<tbody>
<tr>
<td>¹⁷⁸Pt</td>
<td>1.01</td>
<td>-2.36</td>
<td>10.3</td>
<td>-12.47</td>
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<tr>
<td>¹⁸⁰Pt</td>
<td>0.87</td>
<td>2.35</td>
<td>0.06</td>
<td>-0.21</td>
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<tr>
<td>¹⁸²Pt</td>
<td>0.84</td>
<td>7.02</td>
<td>-2.1</td>
<td>1.77</td>
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<tr>
<td>¹⁸⁴Pt</td>
<td>1.006</td>
<td>-5.92</td>
<td>0.83</td>
<td>-0.36</td>
</tr>
<tr>
<td>¹⁸⁶Pt</td>
<td>1.18</td>
<td>-16.51</td>
<td>3.4</td>
<td>-2.16</td>
</tr>
<tr>
<td>¹⁸⁸Pt</td>
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<td>-38.33</td>
<td>12.8</td>
<td>-18.55</td>
</tr>
<tr>
<td>¹⁹⁰Pt</td>
<td>1.77</td>
<td>-0.88</td>
<td>-14.60</td>
<td>30.64</td>
</tr>
<tr>
<td>¹⁸⁰Hg</td>
<td>1.40</td>
<td>-3.18</td>
<td>7.58</td>
<td>-5.83</td>
</tr>
<tr>
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<td>-6.11</td>
<td>18.05</td>
<td>-16.52</td>
</tr>
<tr>
<td>¹⁸⁴Hg</td>
<td>1.31</td>
<td>-2.34</td>
<td>5.28</td>
<td>-3.77</td>
</tr>
<tr>
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<tr>
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<td>-16.24</td>
<td>38.24</td>
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<tr>
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<tr>
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<td>40.81</td>
<td>-31.13</td>
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<tr>
<td>¹⁹⁴Hg</td>
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<td>-18.514</td>
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<tr>
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<td>3.16</td>
<td>-16.55</td>
<td>40.43</td>
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</tr>
<tr>
<td>¹⁹⁸Hg</td>
<td>3.09</td>
<td>-14.60</td>
<td>31.73</td>
<td>-20.99</td>
</tr>
</tbody>
</table>

Fig. (2), represents the relation between the angular momentum \( I \) and \( \Delta E(I)=E(I)-E(I-2) \) for the yrast states of \(^{182-198}\text{Hg}\) isotopes. From the figure it is apparent that the calculated results are in a good agreement with the experimental data.

![Fig. (2)](image)

Fig. (1) The angular momentum \( I \) vs \( \Delta E(I)=E(I)-E(I-2) \) for the yrast states of \(^{178-197}\text{Pt}\) the solid circles represent the experimental data\(^{29-32}\) and the calculated data represents by the open circles.

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**CONCLUSION**

The yrast bands of $^{178-188}$Pt and $^{182-198}$Hg isotopes are studied by a simple model based on the vibrational model with small number of parameters. Generally, the energy levels up to $I=24$ can be reproduced very well. In the heavier $^{190}$Pt the relatively small spacing between the $10^+$ and $12^+$ levels can be satisfactory reproduced by the applied model. Additionally, in the low excitation region of $^{188-198}$Hg isotopes the shape evolution and the closely spaced ($8^+$, $10^+$, $12^+$) triplet structure can be well reproduced by the same model.

**REFERENCES**