Extension of a Dynamic Version of the Collective Model to High Energy States in Even-Mass Ce and Nd Isotopes


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ABSTRACT

The ground state bands in even-even Ce and Nd isotopes were systematically analysed by using a simple modified version of previously proposed approach based on the collective model predictions. The model successfully describes the backbending phenomena in even mass Ce and Nd isotopes and fits remarkably well to the experimental observations with a few parameters.

Keywords: Even Ce and Nd isotopes/ground state bands/ Vibrational Model

INTRODUCTION

In the recent years even-even Ce and Nd nuclei furnish a fertile testing ground for most of the advanced theories, where the calculated properties may be compared with experiments. Previous works showed that there is a good evidence for a major change in the nature of the yrast level below \( I=20 \) in some even-even nuclei and furthermore, that at higher spin values a very regular structure develops. It is simply called the backbending which occurs as one plot the moment of inertia versus the square of the rotational frequency. The main feature of backbending phenomena can be explained in terms of band intersection. The rapid increase of moment of inertia with rotational frequency indicates a major modification of the intrinsic structure at the point where backbending occurs. This fact depends on two factors; the difference between the effective moment of inertia of the interacting bands at their intersection point, which determine the transition to be made and the strength of the interaction between the bands which determine how sharply this transition is made.

Several works have confirmed that backbending could be influenced by the ground state band energy spacing and the pairing gap. Also, the fact that the moment of inertia is almost doubled and is approaching the value of a rigid rotation suggests that the transition is associated with pair correlation. A large amount of work has recently been done in studying the antialignment effect of pairing correlation on the moment of inertia.

Many attempts have been carried out to provide theoretical descriptions of the backbending phenomena. The variable moment of inertia (VMI) gives a very good description of the ground state bands and also \( \beta \)- and \( \gamma \)-bands of even-even nuclei up to the point where backbending occurs. Also, some works were utilized the band mixing calculations to describe backbending which used the same set of VMI parameters for both the ground state and the superband. Other authors used the variable moment of inertia (VMI) for the ground state bands. They assumed the superband to be purely rotation. The two bands are coupled through a general band coupling formula. According to their treatment the energy of the yrast band is given by:

\[
E = \frac{1}{2}(E_s + E_g) + \sqrt{\frac{1}{4}(E_s - E_g)^2 + V^2}.
\]

(1)

Where \( V \) is the interaction potential between the ground state or yrast band \( (E_g) \) and superband \( (E_s) \).

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In this model the levels of the ground state band are described by the standard (VMI) formula

\[ E_i = I(I+1)/2\varphi_o + (1/2)c[\varphi_i - \varphi_o]^2, \]  

Where \( \varphi_o \) is a parameter defined as the “ground-state moment of inertia” and \( c \) is the “restoring force constant”. \( \varphi_i \) is the moment of inertia for each spin \( I \).

For the superband there is a simple possible form, i.e., a purely rotational band characterized by a constant moment of inertia,

\[ E_i(I) = E_y + a[I(I+1) - K^2], \]  

Where \( a \) is the inverse of the constant moment of inertia and \( K \) is a quantum number takes the values 0 or 1.

Several efforts within the framework of the interacting boson approximation model have been attempted to understand the mechanism of backbending. Although satisfactory results have been obtained the calculations employed are generally complex and need extra multiparameters formula.

Further investigation of nuclei in \( A \approx 130 \) region based on the quadrupole moment and angular momentum operators were done in the self-consistent cranked nonrelativistic Hartree-Fock and relativistic Hartree mean-field approaches\(^{14}\) and rather successful results have been obtained. Also, an explanation of backbending based on band crossing suggesting magnetic rotation nature in the mass 130 region has been undertaken.\(^{15}\) The obtained results were supported by theoretical calculations.

The main purpose of the present work is to investigate even-even Ce and Nd nuclei in a phenomenological way in the frame work of a simple four parameter formula based on a dynamic version of the unified collective model. It is hoped by such work to have a good description of the backbending regions besides those of the low-lying states.

**MODEL DESCRIPTION**

The neutron deficient nuclei near \( A \approx 130 \) have the properties of rotational nuclei.\(^{16}\) These nuclei have been found moderately deformed (\( \beta = 0.2 - 0.3 \)) and quite unstable against the deformation. In this region the neutrons and protons both occupy high \( j-h_{11/2} \) orbital; however, the neutron levels are filled up to the mid to high-\( \Omega \) orbital, while the proton Fermi surface is lower in the shell. Thus, these nuclei represent a region in which the neutron-proton pairing effects are not expected to be strong, but where the forces may be modified as compared to isotopes closer to stability. The yrast bands in the heavier (\( A \approx 130 \)) Ce isotopes are characterized by the alignment of a pair of \( h_{11/2} \) protons at spin \( I \approx 10 \hbar \) and rotational frequency \( \omega \approx 0.3 - 0.4 \) MeV/\( \hbar \).\(^{17}\)

An inspection of the energy level spacing of the excited states in neutron deficient nuclei reveals a region of pronounced collective behaviour.\(^{18}\)

A dynamic version of the unified collective model has been previously applied by Zvanov and Mitroshin.\(^{19}\) According to their model, the quasirotational bands of the even-even nuclei can be understood if phonons are considered as dynamic modes whose structure and energy change as their number increases. In such model the energy spectrum of vibrational states with \( I = \lambda N \) is given by

\[ E_y = N\omega^y + \frac{2\lambda + 1}{2}(\omega^y - \alpha'), \]  

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Where $\lambda$ is a constant depends on the number of phonons "$N$" and the spin $I$, for the yrast band $\lambda = 2$, $\omega^N = \omega^1(1 + 2\gamma(N - 1))^{1/2}$, $\omega^1 = E_{2s}$, and $\gamma = \gamma^*\{(1 + 3\omega^2Z^2/10\pi R_{\gamma}E_{2s})\}$ where the value of $\gamma^*$ is a universal constant [$\gamma^* = (0.055(5)-0.00020(5))$] and $B_2 = \frac{R_z^2 A m}{4/3\pi}$.

In the present work, a further improvement of this model is given taking into consideration the possibility that the energy levels of even-even nuclei can be treated as dynamic modes too where the energy $E_N$ can be obtained by the following formula:

$$E_N = A\omega^N + B(\omega^N - \omega^1) + C(\omega^N - \omega^1)^2 + D(\omega^N - \omega^1)^3 + .... \quad (5)$$

Where, $A$, $B$, $C$ and $D$ are constants.

The even power terms in the previous expression are comparable to the so-called Harris expansion$^{20,21}$ for rotational spectra. The odd power terms in Eq.(5) could describe the residual interaction coming from band mixing. Furthermore, Eq.(5) is equivalent to the extended variable moment of inertia model to high spins by Anagnostatos$^{22}$ based on the article given by Das and Banerjee.$^{23}$ In that work$^{22}$ the energy of the state of an even-even nucleus is in the form:

$$E = C_2(\varphi - \varphi_0 )^2 + C_3(\varphi - \varphi_0 )^3 + C_4(\varphi - \varphi_0 )^4 + \frac{I(I+1)}{2\varphi^2} \quad (6)$$

Where $C_2$, $C_3$, $C_4$ and $\varphi_0$ are the four parameters of the model.

From the excitation energy $E(I)$ of the yrast bands one calculates the variable moment of inertia $\varphi$, and the squared rotational frequency $\omega^2$ by using the well-known relations$^{24}$.

$$2\varphi / h^2 = (4I-2) / E_\gamma , \quad (7)$$

and

$$(h\omega)^2 = (I^2 - I + 1)[E_\gamma / (2I - 1)]^2 . \quad (8)$$

Where

$$E_\gamma = E(I) - E(I - 2) . \quad (9)$$

Here we undertake a comparative study between our calculations and the experimental data through $\varphi / \omega^2$ plots.

RESULTS

The yrast levels of the even-even Ce and Nd nuclei up to spin 42 have been studied using our improved modified dynamic version and the obtained results are presented in table 1, together with corresponding experimental energy levels$^{25-29}$. The results of the previously proposed dynamic version and the simple model of backbending are also included in the same table for the sake of comparison.
Table (1). Experimental and calculated energies in (MeV) of ground-state bands in even-even Ce and Nd nuclei, using the simple model (SM) for backbending, the dynamic version model (DVM) and the improved dynamic version model (IDVM).

| Energy | \(E_{cal} | \ E_{exp} |
|--------|------------------|
| 10\(^{4}\)Ce | 18.7018 -29.6786 | 244.1580 | 0.0855 | 6.5250 | 17.6897 | 1.24 | 11.244 | 0.0233 | 0.0059 |
| 11\(^{8}\)Ce | 10.1835 -16.2542 | 203.8391 | 0.0979 | 6.6046 | 14.4865 | 1.11 | 11.824 | 0.0266 | 0.0209 |
| 12\(^{6}\)Ce | 15.7126 -7.2726 | 143.3008 | 0.0396 | 6.0865 | 14.0739 | 1.45 | 11.042 | 0.0348 |
| 13\(^{8}\)Ce | 20.9234 -1.3608 | 7.2382 | 0.0242 | 14.8646 | 7.3314 | 2.61 | 10.1 | 0.0219 | 0.0579 |
| 14\(^{6}\)Nd | 10.1635 -16.2542 | 203.8391 | 0.0979 | 7.5558 | 18.8619 | 1.06 | 11.66 | 0.0463 | 0.0076 |
| 15\(^{6}\)Nd | 13.1559 -11.3241 | 204.6397 | 0.0619 | 5.0865 | 14.0739 | 1.45 | 11.042 | 0.0348 |
| 16\(^{8}\)Nd | 20.1122 -40.7856 | 258.0634 | 0.0893 | 7.6543 | 19.1082 | 1.63 | 12.242 | 0.0188 | 0.0180 |

The predictions of both the present improved model and the successful simple model of backbending\(^{15}\) are to great extent comparable. This result gives our model the superiority because of the relative complexity inherited in the simple model of backbending concerning the determination of the interaction potential. Furthermore, it is clear from table 1 that the predictions of the present improved ground state bands very well the ground-state energy levels in both Ce and Nd even mass isotopes up to high spin.

The calculated parameters are given in table 2 where the root mean square deviation (\(\sigma\)) values of the fitting procedure are also included. The mean square deviation is also given by

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (1 - \frac{E_{cal}}{E_{exp}})^2}.
\]

Table (2). The fitting parameters in (MeV) of the modified dynamic version model and the simple model for backbending.

<table>
<thead>
<tr>
<th>Modified dynamic version</th>
<th>Simple model for backbending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleus</td>
<td>A</td>
</tr>
<tr>
<td>10(^{4})Ce</td>
<td>1</td>
</tr>
<tr>
<td>11(^{8})Ce</td>
<td>1</td>
</tr>
<tr>
<td>12(^{6})Ce</td>
<td>1</td>
</tr>
<tr>
<td>13(^{8})Ce</td>
<td>1</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>16(^{8})Nd</td>
<td>1</td>
</tr>
</tbody>
</table>
In fig. 1 we present the $\phi \omega^2$ plots for the aforementioned nuclei. The backbending displays in fig. 1 shows good agreement between the present model and the experimental data even in the forward regions. Additionally, a satisfactory agreement has been obtained in down-bending region illustrated in the figure concerning $^{128}\text{Ce}$

**Figure. (1).** The moment of inertia $2\phi/\hbar^2$ versus the square of the rotational frequency $(\hbar \omega)^2$ for $^{126-134}\text{Ce}$ and $^{130-134}\text{Nd}$.

**CONCLUSION**

The present study suggests that the modified dynamic version of the collective model gives a fairly accurate description of the high spin states of both Ce and Nd even mass nuclei. Furthermore, the model gives overall satisfactory results concerning the description of the backbending phenomena. Also, it is able to describe well the forward and down-bending regions. In acute backbending cases the model roughly holds so that it needs further microscopic calculations.

**REFERENCES**

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